Nonlinear analysis – Assignment 4

**Part A : Forward Euler method**

Step 1 : One strain increment

An unique strain increment is applied to a non-stressed steel plate.

At first, the stress increment is explicitly derived by the forward Euler method from the strain increment, as following:

* Compute trial stress increment , D being the constitutive matrix, function of Young’s modulus and Poisson ratio of the plate.
* If the trial stress (initial stress + trial stress increment) check the failure condition, the yielding factor is computed, allowing to compute the yield stresses for the same load direction. Similarly, the strain increment is divided in its elastic (until failure condition verified) and plastic parts.
* From its plastic part, the stress is computed as the projection of the trial stress on the tangent of the yield criterion at the yield stress computed previously.
* The criterion value associated with stress is then computed, to observe the “distance“ between the numerical approximation and actual stress update.

For this first step, where no sub-increments are implemented in the stress computation, the following values were obtained:

Step 2 : Sub-increments introduction, 2 sub-increments

The elastic part of the strain increment is divided in two sub-increments. For each sub-increment i, a new is computed, and projected on the tangent of the stress obtained at the previous sub-increment. Therefore, an increased number of sub-increments allows to fit the curvature of the yield criterion, reducing the distance of the numerical stress value.

The first trial stress is kept the same as in step 1. The plastic strain increment is divided by the number of sub-increments, and associated stress sub-increments are computed with the tangent of the previous sub-increment stress.

In a stress x – stress y graph, the forward Euler method for one strain increment with 2 sub-increments is as following:

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Description générée automatiquement

With in the top right corner, and the computed stress update closer to the yield criterion.

For this second step, the following value were obtained:

Step 3 : Minimal number of sub-increments.

We saw in the first two steps that increasing the number of sub-increments reduce the value of the yield criterion, and bring the computed stress update closer to the actual stress update.

Therefore, we want to increase the number of sub-increments in order to have the criterion value under a given threshold, here:

We chose an iterative approach to the problem, calculating for increasing number of sub-increments.

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A minimum of 125 sub-increments was observed in order to obtain a stress update respecting the yield criterion value threshold.

In a stress x – stress y graph, the forward Euler method for one strain increment with 125 sub-increments is as following:

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The following values were obtained:

**Steps results comparison:**

|  |  |  |  |
| --- | --- | --- | --- |
| **Step** | **Number of sub-increments** | **Stress update [Mpa]** | **Criterion value** |
| 1 | 1 |  | 18023 |
| 2 | 2 |  | 4513 |
| 3 | 125 |  | 35.75 |

Firstly, we can observe that the stress sub-increments computations for a large amount of strain sub-increments lie much closer to the yield criterion than for the previous steps. This is expected, as the projection is closer to the previous sub-increment stress, reducing the error.

Also, the stresses varies in a non-negligeable amount compared to the step 1: -15 % for and + 5 % for .

Therefore, increasing the amount of sub-increments is important to a trustworthy result.

Finally, it is observed that the yield criterion value decrease rapidly with the increase of amount of sub-increments, approaching rapidly the limit of 0. Consequently, passed a certain amount, a very large amount of sub-increments will not improve further the stress update value, reaching a limit.